SIMULATION OF LAMINAR NATURAL CONVECTION IN A POROUS CYLINDRICAL ANNULUS WITH DOUBLE-DIFFUSION EFFECTS

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Abstract. *This paper presents results for coupled heat and mass transport under laminar flow regime in a horizontal cylindrical annulus filled with a fluid saturated porous medium. Two driving mechanisms are considered to contribute to the overall momentum transport, namely temperature driven and concentration driven mass fluxes. Aiding and opposing flows are considered, where temperature and concentration gradients are either in the same direction or of different sign, respectively. Modeled equations are presented based on the double-decomposition concept, which considers both time fluctuations and spatial deviations about mean values.*

Keywords*: Double-Diffusion, Porous Media, Natural Convection*

1. INTRODUCTION

The study of double-diffusive natural convection in porous media has many environmental and industrial applications, including grain storage and drying, petrochemical processes, oil and gas extraction, contaminant dispersion in underground water reservoirs, electrochemical processes, etc. The importance of double diffusive natural convection can be better appreciated by the volume of papers published in this field, which was reviewed by Nield and Bejan (1999). The analyses of natural convection in a horizontal cylindrical annuli filled by a porous material has been subject of a number of studies in recent years. Thermal insulators, cryogenics, thermal storage systems, electronic cooling, inert gas insulation of high-voltage electric cables and the determination of the requirements for aircraft cabin insulation.

Accordingly, double diffusive convection in a vertical cavity subject to horizontal temperature gradients has been investigated by Trevisan and Bejan (1985, 1986), Goyeau et al. (1996), Mamou et al. (1995, 1998), Mohamad and Bennacer (2002), Nithiarasu et al. (1997), Bennacer et al. (2001, 2003), among others. In most of the aforementioned papers, the intra-pore flow was assumed to be laminar and it was demonstrated that, depending on the governing parameters of the problem and on the thermal to solute buoyancy ratio, various modes of convection prevail.

The natural convection in cylindrical annular geometry filled with porous material also have been studied by distinct numerical approaches, such as the finite-difference method reported by Caltagirone (1976) and Burns and Tien (1979). Finite element method is found in the work of Motjabi et al. (1987).

This work intends to present a set of macroscopic mass transport equations derived under the recently established double decomposition concept, Pedras and de Lemos (2003), through which the connection between the two paths a) and b) above is unveiled. That methodology, initially developed for the flow variables, has been extended to heat transfer in porous media where both time fluctuations and spatial deviations were considered for velocity and temperature, Rocamora and de Lemos (2000). Buoyant flows, Braga and de Lemos (2004-2009), and mass transfer, de Lemos and Mesquita (2003), have also been investigated. Recently, a general classification of all proposed models for turbulent flow and heat transfer in porous media has been published de Lemos and Pedras (2001).

The purpose of this contribution is to show numerical results for laminar double-diffusive in porous media, which are obtained with the mathematical model earlier proposed in de Lemos and Tofaneli (2004). Here, double-diffusive laminar natural convection flow in porous media is considered.

2. MATHEMATICAL MODEL

The problem considered here is showed schematically in Figure 1a and refers to a concentric annulus completely filled with porous material with outer and inner radii r_0 and r_i, respectively, and $R = r_0/r_i = 2$. The top and bottom walls are kept insulated and the porous medium is considered to be rigid. The binary fluid in the cavity of Figure 1a is assumed to be Newtonian and to satisfy the Boussinesq approximation.

2.1 Local instantaneous transport equation

The steady-state microscopic instantaneous transport equations for an incompressible binary fluid mixture with constant properties are given by:

$$\nabla \cdot \mathbf{u} = 0 \tag{1}$$

$$\rho \nabla \cdot (\mathbf{u}\mathbf{u}) = -\nabla p + \mu \nabla^2 \mathbf{u} + \rho \mathbf{g} \tag{2}$$

$$(\rho c_p) \nabla \cdot (\mathbf{u} T) = \nabla \cdot (\lambda \nabla T) \tag{3}$$

$$\rho \nabla \cdot (\mathbf{u}\, m_\ell + \mathbf{J}_\ell) = \rho\, R_\ell \tag{4}$$

where $\mathbf{u}$ is the mass-averaged velocity of the mixture, $\mathbf{u} = \sum_\ell m_\ell \mathbf{u}_\ell$, $\mathbf{u}_\ell$ is the velocity of species ℓ, m_ℓ is the mass fraction of component ℓ defined as $m_\ell = \rho_\ell / \rho$, ρ_ℓ is the mass density of species ℓ (mass of ℓ over total mixture volume), ρ is the bulk density of the mixture ($\rho = \sum_\ell \rho_\ell$), p is the pressure, μ is the fluid mixture viscosity, $\mathbf{g}$ is the gravity acceleration vector, c_p is the specific heat, T is the temperature and λ is the fluid thermal conductivity. The generation rate of species ℓ per unit of mixture mass is given in Eq. (4) by R_ℓ.

An alternative way of writing the mass transport equation is using the volumetric molar concentration C_ℓ (mol of ℓ over total mixture volume), the molar weight M_ℓ (g/mol of ℓ) and the molar generation/destruction rate R_ℓ^* (mol of ℓ /total mixture volume), giving:

$$M_\ell \nabla \cdot (\mathbf{u}\, C_\ell + \mathbf{J}_\ell) = M_\ell\, R_\ell^* \tag{5}$$

Further, the mass diffusion flux $\mathbf{J}_\ell$ (mass of ℓ per unit area per unit time) in Eq. (4) or Eq. (5) is due to the velocity slip of species ℓ,

$$\mathbf{J} = \rho_\ell (\mathbf{u}_\ell - \mathbf{u}) = -\rho_\ell D_\ell \nabla m_\ell = -M_\ell D_\ell \nabla C_\ell \tag{6}$$

where D_ℓ is the diffusion coefficient of species ℓ into the mixture. The second equality in Eq. (6) is known as Fick's Law, which is a constitutive equation strictly valid for binary mixtures under the absence of any additional driving mechanisms for mass transfer Hsu and Cheng (1990). Therefore, no Soret or Dufour effects are here considered.

Rearranging Eq. (5) for an inert species, dividing it by M_ℓ and dropping the index ℓ for a simple binary mixture, one has,

$$\nabla \cdot (\mathbf{u}\, C) = \nabla \cdot (D \nabla C) \tag{7}$$

If one considers that the density in the last term of Eq. (2) varies with temperature and concentration, for natural convection flow, the Boussinesq hypothesis reads, after renaming this density ρ_T,

$$\rho_T \cong \rho[1 - \beta(T - T_{ref}) - \beta_C (C - C_{ref})] \tag{8}$$

where the subscript ref indicates a reference value and β and β_C are the thermal and salute expansion coefficients, respectively, defined by,

$$\beta = -\frac{1}{\rho}\frac{\partial \rho}{\partial T}\bigg|_{p,C}, \quad \beta_C = -\frac{1}{\rho}\frac{\partial \rho}{\partial C}\bigg|_{p,T} \tag{9}$$

Equation (8) is an approximation of Eq. (9) and shows how density varies with temperature and concentration in the body force term of the momentum equation.

Further, substituting Eq. (8) into Eq. (2), one has,

$$\rho \nabla \cdot (\mathbf{u}\mathbf{u}) = -\nabla p + \mu \nabla^2 \mathbf{u} + \rho \mathbf{g}[1 - \beta (T - T_{ref}) - \beta_C (C - C_{ref})] \tag{10}$$

Thus, the momentum equation becomes,

$$\rho\nabla\cdot(\mathbf{uu}) = -(\nabla p)^{*} + \mu\nabla^{2}\mathbf{u} - \rho\mathbf{g}[(\beta(T-T_{ref}) + \beta_{C}(C-C_{ref})] \tag{11}$$

where $(\nabla p)^{*} = \nabla p - \rho\mathbf{g}$ is a modified pressure gradient.

As mentioned, there are, in principle, two ways that one can follow in order to treat turbulent flow in porous media. The first method applies a time average operator to the governing Eq. (4) before the volume average procedure is conducted. In the second approach, the order of application of the two average operators is reversed. Both techniques aim at derivation of a suitable macroscopic turbulent mass transport equation.

Volume averaging in a porous medium, described in detail in references Slattery (1967), Whitaker (1969) and Gray and Lee (1972), makes use of the concept of a Representative Elementary Volume (REV), over which local equations are integrated. After integration, detailed information within the volume is lost and, instead of, overall properties referring to a REV are considered. In a similar manner, statistical analysis of turbulent flow leads to time mean properties. Transport equations for statistical values are considered in lieu of instantaneous information on the flow.

Before undertaking the task of developing macroscopic equations, it is convenient to recall the definition of volume average.

2.2 Volume average operator

The volume average of φ taken over a Representative Elementary Volume in a porous medium can be written as:

$$\langle\varphi\rangle^{v} = \frac{1}{\Delta V}\int_{\Delta V}\varphi\, dV \tag{12}$$

The value $\langle\varphi\rangle^{v}$ is defined for any point x surrounded by a Representative Elementary Volume, of size ΔV. This average is related to the intrinsic average for the fluid phase as:

$$\langle\varphi_{f}\rangle^{v} = \phi\langle\varphi_{f}\rangle^{i} \tag{13}$$

where $\phi = \Delta V_{f}/\Delta V$ is the medium porosity and ΔV_{f} is the volume occupied by the fluid in a REV. Furthermore, one can write:

$$\varphi = \langle\varphi\rangle^{i} + {}^{i}\varphi \tag{14}$$

with $\langle {}^{i}\varphi\rangle^{i} = 0$. In Eq. (14), ${}^{i}\varphi$ is the spatial deviation of φ with respect to the intrinsic average $\langle\varphi\rangle^{i}$.

Further, the local volume average theorem can be expressed as Slattery (1967), Whitaker (1969) and Gray and Lee (1972):

$$\begin{aligned}
\langle\nabla\varphi\rangle^{v} &= \nabla(\phi\langle\varphi\rangle^{i}) + \frac{1}{\Delta V}\int_{A_i}\mathbf{n}\varphi\, dS \\
\langle\nabla\cdot\boldsymbol{\varphi}\rangle^{v} &= \nabla\cdot(\phi\langle\boldsymbol{\varphi}\rangle^{i}) + \frac{1}{\Delta V}\int_{A_i}\mathbf{n}\cdot\boldsymbol{\varphi}\, dS \\
\langle\frac{\partial\varphi}{\partial t}\rangle^{v} &= \frac{\partial}{\partial t}(\phi\langle\varphi\rangle^{i}) - \frac{1}{\Delta V}\int_{A_i}\mathbf{n}\cdot(\mathbf{u}_{i}\varphi)dS
\end{aligned} \tag{15}$$

where A_i and $\mathbf{u}_i$ represent the area and velocity of the interface fluid/solid, respectively, and $\mathbf{n}$ is the external unit vector to the fluid and normal to the A_i. It is important to emphasize that Ai should not be confused with the surface area surrounding volume ΔV.

2.3 Macroscopic equations for buoyancy free flows

For non-buoyant flows, macroscopic equations considering turbulence have been already derived in detail for momentum Pedras and de Lemos (2001), heat de Lemos and Braga (2003) and mass de Lemos and Mesquita (2003) transfer and for this reason their derivation need not to be repeated here. They read:

Momentum transport

$$\rho\nabla\cdot\left(\frac{\mathbf{u}_D\mathbf{u}_D}{\phi}\right)=-\nabla(\phi\langle p\rangle^i)+\mu\nabla^2\mathbf{u}_D-\left[\frac{\mu\phi}{K}\mathbf{u}_D+\frac{c_F\phi\rho|\mathbf{u}_D|\,\mathbf{u}_D}{\sqrt{K}}\right] \tag{16}$$

Heat transport

$$(\rho c_p)_f\,\nabla\cdot(\mathbf{u}_D\langle T\rangle^i)=\nabla\cdot\{\mathbf{K}_{eff}\cdot\nabla\langle T\rangle^i\} \tag{17}$$

$$\mathbf{K}_{eff}=[\phi\lambda_f+(1-\phi)\lambda_s]\mathbf{I}+\mathbf{K}_{tor}+\mathbf{K}_{disp} \tag{18}$$

The subscripts f and s refer to fluid and solid phases, respectively, and coefficients **K**'s come from the modeling of the following mechanisms:

- Tortuosity: $\left[\frac{1}{\Delta V}\int_{A_i}\mathbf{n}\left(\lambda_f T_f-\lambda_s T_s\right)dS\right]=\mathbf{K}_{tor}\cdot\nabla\langle T\rangle^i$ (19)

- Thermal Dispersion: $-(\rho c_p)_f\,\phi\langle^i\mathbf{u}^i T_f\rangle^i=\mathbf{K}_{disp}\cdot\nabla\langle T\rangle^i$ (20)

Mass transport

$$\nabla\cdot(\mathbf{u}_D\langle C\rangle^i)=\nabla\cdot\mathbf{D}_{eff}\cdot\nabla(\phi\langle C\rangle^i) \tag{21}$$

$$\mathbf{D}_{eff}=\mathbf{D}_{disp}+\mathbf{D}_{diff} \tag{22}$$

$$\mathbf{D}_{diff}=\langle D\rangle^i\mathbf{I}=\frac{1}{\rho}\frac{\mu_\phi}{Sc}\mathbf{I} \tag{23}$$

The coefficients $\mathbf{D}_{disp}$ in Eq. (21) appear due to the nonlinearity of the convection term. They come from the modeling of the following mechanisms:

- Mass dispersion: $-\langle^i\mathbf{u}^{\,i}C\rangle^i=\mathbf{D}_{disp}\cdot\nabla\langle C\rangle^i$ (24)

2.4 Macroscopic double-diffusion effects

Focusing now attention to buoyancy effects only, application of the volume average procedure to the last term of Eq. (11) leads to,

$$\langle\rho\mathbf{g}[\beta(T-T_{ref})+\beta_C(C-C_{ref})]\rangle^v=\frac{\Delta V_f}{\Delta V}\frac{1}{\Delta V_f}\int_{\Delta V_f}\rho\mathbf{g}[\beta(T-T_{ref})+\beta_C(C-C_{ref})]dV \tag{25}$$

Expanding the left hand side of Eq. (25) in light of Eq. (14), the buoyancy term becomes,

$$\begin{aligned}&\langle \rho \mathbf{g}[\beta (T - T_{ref}) + \beta_C (C - C_{ref})]\rangle^v \\ &= \rho \mathbf{g} \phi [\beta_\phi (\langle T \rangle^i - T_{ref}) + \beta_{C_\phi} (\langle C \rangle^i - C_{ref})] + \underbrace{\rho \mathbf{g} \beta \phi \left\langle {}^i T \right\rangle^i}_{=0} + \underbrace{\rho \mathbf{g} \beta_C \phi \left\langle {}^i C \right\rangle^i}_{=0}\end{aligned} \tag{26}$$

where the third and forth terms on the r.h.s. are null since $\langle {}^i \varphi \rangle^i = 0$. Here, coefficients β_ϕ and $\beta_{C\phi}$ are the macroscopic thermal and salute expansion coefficients, respectively. Assuming that gravity is constant over the REV, expressions for them based on Eq. (26) are given as,

$$\beta_\phi = \frac{\langle \rho \beta (T - T_{ref}) \rangle^v}{\rho \phi (\langle T \rangle^i - T_{ref})} ; \beta_{C_\phi} = \frac{\langle \rho \beta_C (C - C_{ref}) \rangle^v}{\rho \phi (\langle C \rangle^i - C_{ref})} \tag{27}$$

Including Eq. (26) into Eq. (16), the macroscopic time-mean Navier-Stokes (NS) equation for an incompressible fluid with constant properties is given as,

$$\begin{aligned}\rho \nabla \cdot \left(\frac{\mathbf{u}_D \mathbf{u}_D}{\phi} \right) &= -\nabla (\phi \langle p \rangle^i) + \mu \nabla^2 \mathbf{u}_D + \rho \mathbf{g} \phi \left[\beta_\phi (\langle T \rangle^i - T_{ref}) + \beta_{C_\phi} (\langle C \rangle^i - C_{ref}) \right] \\ &- \left[\frac{\mu \phi}{K} \mathbf{u}_D + \frac{c_F \phi \rho |\mathbf{u}_D| \, \mathbf{u}_D}{\sqrt{K}} \right]\end{aligned} \tag{28}$$

Coefficients β_ϕ and β_{C_ϕ} are used to compose the Grashof numbers associated with the thermal and solute drives, in the form,

$$Gr_\phi = \frac{g \beta_\phi \Delta T H^3}{\nu^2}, Gr_{C_\phi} = \frac{g \beta_{C_\phi} \Delta C H^3}{\nu^2} \tag{29}$$

where $\Delta T = T_1 - T_2$ and $\Delta C = C_1 - C_2$ are the maximum temperature and concentration variation across the cavity, respectively. One should note that for opposing thermal and concentrations drives, such maximum differences are of opposing signs.

The ratio of Grashof numbers define the buoyancy ratio N in the form

$$N = \frac{Gr_{C_\phi}}{Gr_\phi} = \frac{\beta_{C_\phi} \Delta C}{\beta_\phi \Delta T} \tag{30}$$

giving for Eq. (28),

$$\begin{aligned}\rho \nabla \cdot \left(\frac{\mathbf{u}_D \mathbf{u}_D}{\phi} \right) &= -\nabla (\phi \langle p \rangle^i) + \mu \nabla^2 \mathbf{u}_D + \rho \mathbf{g} \phi \beta_\phi \left\{ (\langle T \rangle^i - T_{ref}) + N \frac{\Delta C}{\Delta T} (\langle C \rangle^i - C_{ref}) \right\} \\ &- \left[\frac{\mu \phi}{K} \mathbf{u}_D + \frac{c_F \phi \rho |\mathbf{u}_D| \, \mathbf{u}_D}{\sqrt{K}} \right]\end{aligned} \tag{31}$$

Either $\beta_{C_\phi} = 0$ or $\Delta C = 0$ results in $N = 0$, or say, only thermal drive applies in Eq. (31). Also, for $\beta_{C_\phi} = 0$ and $\Delta C \neq 0$ in Eq. (31), although no concentration drive is considered, a distribution of C within the field will occur due to the flow established by the thermal drive.

2.5 Integral parameter

The local Nusselt number on the heated inner cylinder for the horizontal cylindrical annuli considering half domain is given by,

$$Nu = -\ln R\left(r\frac{\partial \langle T\rangle^{i}}{\partial r}\right)_{r=r_i} \tag{32}$$

The average Nusselt number is then given by,

$$\overline{Nu} = -\frac{\ln R}{\pi}\int_0^{\pi}\left(r\frac{\partial \langle T\rangle^{i}}{\partial r}\right)_{r=r_i} d\theta \tag{33}$$

3. NUMERICAL DETAILS

The numerical method employed for discretizing the governing equations is the control-volume approach. A hybrid scheme, which includes both the Upwind Differencing Scheme (UDS) and the Central Differencing Scheme (CDS), was used for interpolating the convection fluxes. The well-established SIMPLE algorithm Patankar and Spalding (1972), was followed for handling the pressure-velocity coupling. Individual algebraic equations sets were solved by the SIP procedure of Stone (1968). In addition, concentration of nodal points closer to the walls reduces eventual errors due to numerical diffusion which, in turn, are further annihilated due to the hybrid scheme here adopted. Calculations for laminar and turbulent flows used a 50×50 stretched grid for all cases (Fig. 1b). For turbulent flow calculations, wall log laws were applied.

4. RESULTS AND CONCLUSIONS

The problem considered is showed schematically in Fig. 1 and refers to a concentric annulus completely filled with porous material with outer and inner radii r_0 and r_i, respectively, and $R = r_0/r_i = 2$. The cavity is isothermally heated from the inner cylinder and cooled from outer cylinder, with $T_1 > T_2$ and $C_1 > C_2$. The Rayleigh number is defined as $Ra_m = g\beta_\phi\left(\rho c_p\right)_f \Delta T K r_i \left/ k_{eff}\nu_f\right.$. As in the case of a square cavity filled with porous material, the parameters (Prandtl number, inertia parameter, conductivity ratio) are fixed.

Many workers have focused their attention on the bifurcation and stability of the numerical solution. This work has not this intention and its objective is to validate the numerical tool comparing the present results with others numerical and experimental works, except that only considering the thermal buoyancy, i.e., $N = 0$. Here these results were extended taking into account the thrust and thermal mass acting simultaneously.

According to Caltagirone (1976), there are three convection regimes. The first is for $Ra^* \le 8$ and the convective phenomena are very little developed and the heat transfer occurs only by conduction. This regime will be called pseudo-conduction. The second one is in the interval where the $8 < Ra^* < 65$ and the convective phenomena are found to be steady. The fluids warm up on contact with the inner cylinder and fall along the outer surface. The last regime is for $Ra > 65$ where a new type of evolution appears. Perturbations occur in the upper part of the annular layer, and are shown by fluctuations in temperature.

Figure 2 shows the isotherms and streamlines of a concentric annuli heated from the inner cylinder and cooled from outer cylinder completely filled with porous material for $Ra = 25$, $Ra = 200$ and $r_0/r_1 = 2$, for buoyancy ratio, $N = 0$. The figure show a good agreement with the work of Caltagirone (1976) and reproduce the basic features of the flow. Figure 3 shows the isotherms and streamlines for the same Rayleigh numbers in Fig. 2, but here the value of the buoyancy ratio is $N = 1$, we can observe that the behavior of these lines compared to $N = 0$, has almost the same behavior.

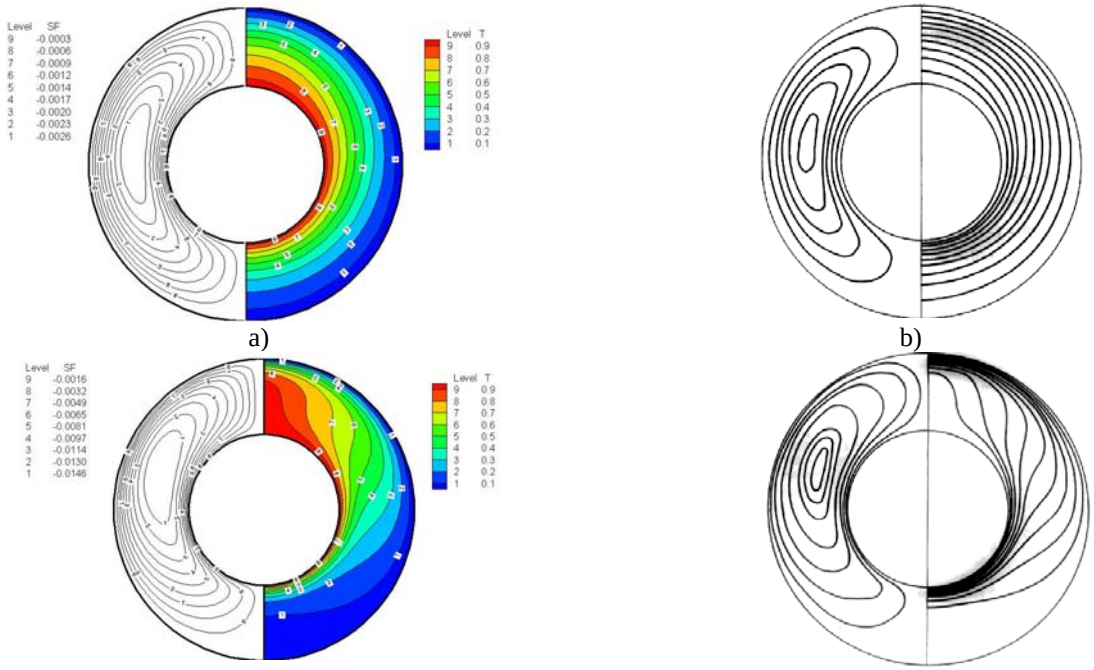
Table 1 shows, for some Rayleigh numbers, the average Nusselt number $\overline{Nu}$ on the heated inner cylinder. It is seen from Tab. 1 that computations fall within the range of values presented in the literature taking into account only the thermal buoyancy in the case of the present work $N = 0$. Again, it is seen from Tab.1 that the agreement between the present and previous results is reasonable.

The heat transfer coefficient is seen to increase with Rayleigh number distorting the isotherms as convection becomes dominant, but the streamlines do not present such intense variations, see Fig. 2a) and Fig. 2c). We also observed that there is a small difference when considering the buoyancy ratio $N = 1$.

Is important to emphasize, that the Darcy number was considered as a constant and several runs were performed for different permeabilities. Although not shown here, one observes that the lower the permeability, the higher the average Nusselt number. In comparison with results of Tab. 1, more accurated simulations were obtained for low permeability media.

a) b)

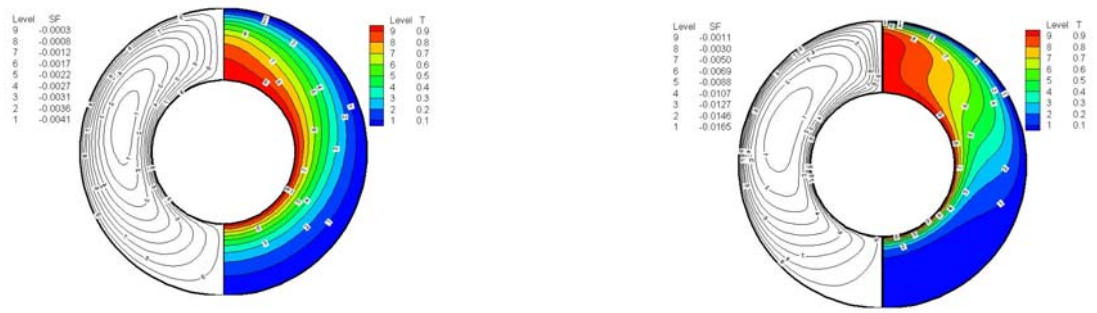
Figure 1. Schematic of the problem: a) geometry; b) grid.



a) b)

c) d)

Figure 2. Laminar Isotherms and Streamlines for $Ra_m = 25$ and $Ra_m = 200$ with $\phi = 0{,}2$, $D_p = 3mm$ and $r_0/r_i = 2$; a), c) Presents Results, $N = 0$; b), d) Caltagirone (1976), ($N = 0$).



a) b)

Figure 3. Laminar Isotherms and Streamlines for $Ra_m = 25$ and $Ra_m = 200$ with $\phi = 0{,}2$, $D_p = 3mm$ and $r_0/r_i = 2$ with $N = 1$.

Table 1. Average Nusselt Number for Ra_m ranging from 25 to 500 with $K = 0{,}7813\times10^{-9}m^2$, $D_p = 3mm$, $Da = 3{,}125\times10^{-7}$, $k_s/k_f = 1$, $\phi = 0{,}2$, $\Pr = 7$.

Ra_m					
	25	***100***	***150***	***200***	***500***
Caltagirone (1976)	1,0993	1,8286	-	2,6256	4,1983
Charrier-Mojtai (1997)	-	1,8670	2,3090	-	-
Braga (2006)	1,1095	1,8629	2,3023	2,6764	4,2741
Present Results, with $N = 0$	1,1076	1,8544	2,3022	2,6495	4,2631
Present Results, with $N = 1$	1,1193	2,2665	2,7744	3,1975	4,9405

5. ACKNOWLEDGEMENTS

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